

# Collective effects of multi-scatterer on coherent propagation of photon in a two dimensional network

D. Z. Xu<sup>1</sup>, Yong Li<sup>2</sup>, C. P. Sun<sup>2</sup>, and Peng Zhang<sup>3\*</sup>

<sup>1</sup>*State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Science and University of the Chinese Academy of Sciences, Beijing 100190, China*

<sup>2</sup>*Beijing Computational Science Research Center, Beijing 100084, China*

<sup>3</sup>*Department of Physics, Renmin University of China, Beijing 100872, China*

We study the collective phenomenon in the scattering of a single-photon by one or two layers of two-level atoms. By modeling the photon dispersion with a two-dimensional (2D) coupled cavity array, we analytically derive the scattering probability of a single-photon. It is discovered that in the case with one layer of atoms, the atomic collective excitation leads to a shift for the single-photon scattering spectrum. Such a shift is related to the density of the atomic ensemble. For the case with two layers of atoms, an inter-layer effective coupling appears and induces an electromagnetic-induced-transparency-like phenomenon for the single-photon scattering. Our result provides a new scheme of analyzing photon coherent transport in 2D and may help to understand the current experiments about the high energy photon scattering by the layer nuclei material.

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## I. INTRODUCTION

For the purpose of controlling the transport and the scattering of a single-photon in a full quantum fashion, progress has been made in confined one-dimension systems[1–3]. The transport of a single-photon can be coherently manipulated by the interaction between the photon and the doped natural or artificial atoms. The corresponding physical implementations could be realized in several different ways, such as: defected photonic crystals[4–8], superconducting transmission line resonators [9–12] and coupled cavity arrays (CCA) [13–17]. By modeling the photon hopping between the cavities as a tight-binding Hamiltonian, the photon in the cavity array displays a nonlinear dispersion relation. This fact makes the cavity array a well-controllable platform for studying the photon transport with different quantum natures. For example, with a tunable two-level atom inside one of the cavities, the transmission and reflection of a single-photon can be well controlled in one-dimensional CCA [14].

In order to fabricate integrated all-optical on-chip devices, it is necessary to move forward to the two-dimensional (2D) structures. Based on CCA systems, a lot of theoretical investigations have been proposed to use this system as an alternative for quantum simulation of many body physics [18–22]. However, the studies of coherent scattering of a single-photon in this 2D cavity array is still lack to our best knowledge.

In this paper, we generally study the scattering problem of a single-photon in the 2D cavity array with two-level atoms. In our model, the atoms are periodically located in one or two rows of the cavities. We analytically solve the scattering problem, and obtain the exact

expression for the single-photon scattering probability. These results explicitly display the collective coherent effects in the scattering process. For the case with one layer of atoms, we find that the photon-atom interaction induces a collective energy shift for the atom, and the scattering probability takes the maximum value when the incident photon is resonant with the shifted atomic energy. In the presence of two layers of atoms, we find that the effective couplings between atoms in different layers are induced by the photon-atom interaction. As a result, the atoms are in two dressed states with different effective energies, and the maximum of the scattering probability appears when the incident photon is resonant with one of these two states. Similar as the atomic susceptibility in the system with electromagnetic induced transparency (EIT), the scattering probability has a double-peak behavior as a function of photon-atom detuning. Therefore, the single-photon scattering in 2D-CCA implies more abundant phenomenon than in the 1D case, and this simple model may offer us an intuitional microscopic understanding of the recent experiments of X-ray quantum optics [23, 24].

The rest of the paper is organized as follows. In Sec. II we investigate the single-photon scattering in a 2D-CCA with one layer of atoms, and discuss the collective shift of the atomic energy. In Sec. III we consider the case with two layers of atoms, and illustrate the EIT-like behavior of the scattering probability. There are some conclusion and discussion in Sec. IV. Some detail of our calculation are presented in the appendix.

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\*Electronic address: pengzhang@ruc.edu.cn

## II. SINGLE-PHOTON SCATTERING WITH ONE LAYER OF ATOMS

### A. System and Hamiltonian

We consider a 2D array of identical single-mode cavities as shown in Fig. 1. We further assume that the photons can hop between neighbor cavities. Then the cavity array is described by a tight-binding model as

$$H_c = \sum_{x,y=-\infty}^{+\infty} \omega_c a_{(x,y)}^\dagger a_{(x,y)} - \xi [a_{(x+1,y)}^\dagger a_{(x,y)} + a_{(x,y+1)}^\dagger a_{(x,y)} + h.c.]. \quad (1)$$

Here  $\omega_c$  is the frequency of the photons in the cavities,  $\xi$  is the hopping intensity or the inter-cavity coupling strength,  $a_{(x,y)}$  and  $a_{(x,y)}^\dagger$  are the annihilation and creation operators of the photon in the cavity at position  $(x,y)$ , respectively. Here and after we set  $\hbar = 1$ . When there is a single photon propagating in the system, the eigenstate  $|\vec{k}\rangle$  of  $H_c$  takes the form of 2D plane wave with momentum  $\vec{k} = (k_x, k_y)$ :

$$|\vec{k}\rangle = \frac{1}{2\pi} \sum_{x,y=-\infty}^{+\infty} e^{i(k_x x + k_y y)} a_{(x,y)}^\dagger |\text{vac}\rangle \quad (2)$$

with  $|\text{vac}\rangle$  the vacuum state of all the cavities. The single-photon dispersive relation of the 2D cavity array

$$\epsilon_{\vec{k}} \equiv \epsilon_{(k_x, k_y)} = \omega_c - 2\xi(\cos k_x + \cos k_y) \quad (3)$$

is naturally obtained by the stationary Schrödinger equation  $H_c |\vec{k}\rangle = \epsilon_{\vec{k}} |\vec{k}\rangle$ .

To explore the scattering character of the incident photon on a collection of identical atoms with geometrical configuration, we embed a two-level atom in every  $d$  cavity along the  $y$ -axis, and the  $j$ -th atom is in the cavity at  $(0, dj)$ . The Hamiltonian of these atoms is

$$H_a = \omega_a \sum_j |e\rangle_j \langle e|, \quad (4)$$

with  $\omega_a$  the energy-level spacing of the two-level atom and  $|e\rangle_j$  ( $|g\rangle_j$ ) the excited (ground) state of the  $j$ -th atom. The atom-photon coupling is described by the Jaynes-Cummings Hamiltonian

$$V = \sum_j (\Omega a_{(0,dj)} |e\rangle_j \langle g| + h.c.), \quad (5)$$

with  $\Omega$  the coupling strength.

The setup studied in this paper can be physically implemented by placing the two-level atoms in the defected cavities of the 2D optical crystals [18]. At the match condition  $(k_{x(y)} \sim \pi/2)$ , the energy  $\epsilon_{\vec{k}}$  becomes a linear function of the momentum  $\vec{k}$  as  $\epsilon_{\vec{k}} \approx \omega_c + 2\xi(k_x + k_y)$ . Therefore, in that region our model can also characterize the scattering of photons on the array of atoms in the free space, e.g., the scattering process in the recent nuclear quantum optical experiments with X-ray.

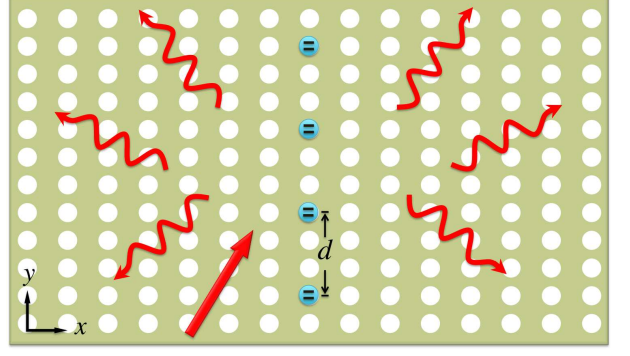


Figure 1: (Color online) The single-photon scattering by one layer of atoms in a 2D CCA. The two-level atoms are confined in one row of the array with periodic  $d$  (in the figure we show the case with  $d = 3$ ). During the scattering process, the incident photon (solid line) can be scattered in several different outgoing directions (waved lines).

### B. single-photon scattering state and $T$ -matrix

Now we calculate the scattering probability of a single-photon scattered by the two-level atoms in our system. To this end, we first derive the single-photon scattering state and the on-shell element of the  $T$ -matrix in this subsection. With the help of these results, we will obtain the single-photon scattering probability in the next subsection.

The scattering state  $|\Psi^{(+)}\rangle$  is given by the Lippman-Schwinger equation

$$|\Psi^{(+)}\rangle = |\vec{k}\rangle |\tilde{g}\rangle + \frac{1}{\epsilon_{\vec{k}} - (H_a + H_c) + i0^+} V |\Psi^{(+)}\rangle \quad (6)$$

with  $|\tilde{g}\rangle \equiv \prod_j |g\rangle_j$  the collective ground state of all atoms. It is clear that in our system the total excitation  $\sum_{x,y=-\infty}^{+\infty} a_{(x,y)}^\dagger a_{(x,y)} + \sum_j |e\rangle_j \langle e|$  is conserved. Thus, in the subspace with one excitation we can expand the stationary eigenstate as

$$|\Psi^{(+)}\rangle = |\phi\rangle |\tilde{g}\rangle + \sum_j \beta_j |\text{vac}\rangle |\tilde{e}_j\rangle. \quad (7)$$

Here  $|\phi\rangle$  is a single-photon state of the cavity modes, and  $|\tilde{e}_j\rangle$  is defined as  $|\tilde{e}_j\rangle \equiv |e\rangle_j \otimes \prod_{l \neq j} |g\rangle_l$  which represents the state with only the  $j$ -th atom excited. It follows from Eq.(6) and Eq.(7) that  $|\phi\rangle$  and  $\beta_j$  satisfy

$$|\phi\rangle = |\vec{k}\rangle + \sum_j \frac{\Omega^* \beta_j}{\epsilon_{\vec{k}} - \omega_c + i0^+} a_{(0,dj)}^\dagger |\text{vac}\rangle, \quad (8)$$

$$\beta_j = \frac{\Omega}{\epsilon_{\vec{k}} - \omega_a} \langle \text{vac} | a_{(0,dj)} | \phi \rangle. \quad (9)$$

Eqs. (8, 9) can be analytically solved by the following approach. First, we notice that our system is invariant under the translation for  $d$  cavities along the

$y$ -axis. Namely, the total Hamiltonian  $H_a + H_c + V$  of our system is commutative with the translation operator  $D$ , which is defined as  $Da_{(x,y)}D^\dagger = a_{(x,y+d)}$ ,  $D|\tilde{e}_j\rangle\langle\tilde{e}_j|D^\dagger = |\tilde{e}_{j+1}\rangle\langle\tilde{e}_{j+1}|$ , and  $D|\tilde{g}\rangle\langle\tilde{g}|D^\dagger = |\tilde{g}\rangle\langle\tilde{g}|$ . As a result of this symmetry, the scattering state  $|\Psi^{(+)}\rangle$  in Eq. (6) satisfies  $D|\Psi^{(+)}\rangle = \exp(ik_y d)|\Psi^{(+)}\rangle$ . Therefore, we can conclude that coefficients  $\beta_j$  takes the form

$$\beta_j = \beta e^{ik_y dj}. \quad (10)$$

Second, substituting Eq. (10) into Eqs. (8, 9), we obtain the expression for the  $j$ -independent coefficient  $\beta$ :

$$\beta = \frac{\Omega}{2\pi} \frac{1}{\epsilon_{\vec{k}} - \omega_a - \Sigma(\vec{k})}, \quad (11)$$

where the self-energy  $\Sigma(\vec{k})$  is given by

$$\Sigma(\vec{k}) = \sum_{l=0}^{d-1} \Sigma_l(\vec{k}). \quad (12)$$

Here the function  $\Sigma_l(\vec{k})$  is defined as

$$\Sigma_l(\vec{k}) = \frac{|\Omega|^2}{2\pi d} \int_{-\pi}^{\pi} dq_x \frac{1}{\epsilon_{\vec{k}} - \epsilon_{[q_x, p_l(k_y)]} + i0^+}. \quad (13)$$

with

$$p_l(k_y) \equiv (k_y + \pi + \frac{2\pi|l|}{d}) \bmod[2\pi] - \pi. \quad (14)$$

It is pointed out that, in our system the self-energy  $\Sigma(\vec{k})$  takes finite value (except for some special momentums which will be discussed later) and thus the renormalization technique is not required. That is due to the fact that the single-photon energy  $\epsilon_{\vec{k}}$  has a finite upper limit  $\omega_c + 2\xi$ . To obtain Eq.(13) we have also used the formula

$$\sum_{j=-\infty}^{+\infty} e^{-i(q_y - k_y)dj} = \frac{2\pi}{d} \sum_{l=0}^{d-1} \delta[q_y - p_l(k_y)], \quad (15)$$

for  $q_y \in [-\pi, \pi]$ .

Furthermore, we can treat the the integral in  $\Sigma_l(\vec{k})$  analytically and get the result

$$\Sigma_l(\vec{k}) = \begin{cases} -i|\Omega|^2 \left[ 2d\xi \sqrt{1 - A_l^2} \right]^{-1}, & |A_l| < 1, \\ -\text{sign}(A_l) |\Omega|^2 \left[ 2d\xi \sqrt{A_l^2 - 1} \right]^{-1}, & |A_l| > 1, \end{cases}, \quad (16)$$

with  $A_l$  defined as

$$A_l \equiv \cos k_x + \cos k_y - \cos[p_l(k_y)]. \quad (17)$$

Substituting the results (16, 11) into Eqs. (10, 8, 7), we finally obtain the analytical expressions of the state  $|\phi\rangle$ , the coefficient  $\beta_j$ , and the scattering state  $|\Psi^{(+)}\rangle$ .

With the analytical expression of the scattering state, we can calculate the on-shell element  $t(\vec{k}' \leftarrow \vec{k})$  of the  $T$ -matrix. According to the scattering theory,  $t(\vec{k}' \leftarrow \vec{k})$  is defined as  $t(\vec{k}' \leftarrow \vec{k}) = \langle \tilde{g} | \langle \vec{k}' | V | \Psi^{(+)} \rangle$ . The straightforward calculation yields

$$t(\vec{k}' \leftarrow \vec{k}) = u_1(\vec{k}) \sum_{l=0}^{d-1} \delta[k'_y - p_l(k_y)], \quad (18)$$

where  $\vec{k}' = (k'_x, k'_y)$  and the function  $u_1(\vec{k})$  is defined as

$$u_1(\vec{k}) = \frac{|\Omega|^2}{2\pi d \left[ \Delta - 2\xi (\cos k_x + \cos k_y) - \Sigma(\vec{k}) \right]}. \quad (19)$$

Here the photon-atom detuning  $\Delta$  is defined as

$$\Delta = \omega_c - \omega_a. \quad (20)$$

Due to the delta functions in Eq. (18), the  $y$ -component  $k'_y$  of the outgoing momentum can only take  $d$  possible values. That is also due to the translation symmetry along the  $y$ -axis of our system.

### C. Single-photon scattering probability

Using the above results of the  $T$ -matrix element, we can calculate the single-photon scattering probability. To this end, we consider the scattering of a single-photon wave packet on the atoms. In the scattering process, the incident wave packet of the photon can be expressed as

$$|\Phi^{(in)}\rangle = \int d\vec{k} \phi^{(in)}(\vec{k}) |\vec{k}\rangle |\tilde{g}\rangle. \quad (21)$$

Here  $\phi^{(in)}(\vec{k})$  is single-photon wave function in the momentum representation and satisfies  $\int d\vec{k} |\phi^{(in)}(\vec{k})|^2 = 1$ . We further assume  $\phi^{(in)}(\vec{k})$  sharply peaks at a specific momentum  $\vec{k}_0 = (k_{0x}, k_{0y})$ . According the scattering theory, when the scattering process is completed, the single-photon state can be expressed as

$$|\Phi^{(out)}\rangle = \int d\vec{k} |\vec{k}\rangle |\tilde{g}\rangle \langle \tilde{g} | \langle \vec{k} | S | \Phi^{(in)} \rangle \quad (22)$$

$$\equiv \int d\vec{k} \phi^{(out)}(\vec{k}) |\vec{k}\rangle |\tilde{g}\rangle \quad (23)$$

in the interaction picture. Here the  $S$ -matrix satisfies

$$\langle \tilde{g} | \langle \vec{k}' | S | \vec{k} \rangle |\tilde{g}\rangle = \delta(\vec{k}' - \vec{k}) - 2\pi i \delta(\epsilon_{\vec{k}} - \epsilon_{\vec{k}'}) t(\vec{k}' \leftarrow \vec{k}). \quad (24)$$

Substituting Eqs. (18, 24) into Eqs. (22, 23), it is easy to find that after the scattering process, the incident wave packet splits into  $(2d - 1)$  different ones (see Appendix). Namely, the out-put wave function in Eq. (23) is given by

$$\phi^{(out)}(\vec{k}) = \sum_{l=-(d-1)}^{d-1} \phi_l^{(out)}(\vec{k}). \quad (25)$$

Here the  $l$ -th wave packet  $\phi_l^{(out)}(\vec{k})$  sharply peaks at a momentum  $\vec{k}_l = [k_{lx}, p_l(k_{0y})]$  which satisfies  $\epsilon_{\vec{k}_l} = \epsilon_{\vec{k}_0}$  and  $\text{sign}(k_{lx}) = \text{sign}(l)$  (see Appendix).

Then the probability for the photon being scattered to the  $l$ -th ( $l \neq 0$ ) outgoing momentum  $\vec{k}_l$  is  $P_l = \int d\vec{k} |\phi_l^{(out)}(\vec{k})|^2$ . As shown in the Appendix,  $P_l$  can be expressed as

$$P_l = \frac{|u_l(\vec{k}_0)|^2}{4\xi^2 |\sin k_{0x} \sin k_{lx}|}. \quad (26)$$

Therefore, for an incident photon with central momentum  $\vec{k}_0$ , the scattering probability  $R_I(\vec{k}_0)$  is

$$R_I(\vec{k}_0) = \sum_{l=-(d-1)}^{d-1} P_l = 2 \sum_{l=1}^{d-1} P_l + P_0, \quad (27)$$

where we have used the fact  $P_l = P_{-l}$ .

#### D. Behavior of the scattering probability

Now we discuss the behavior of the scattering probability  $R_I(\vec{k})$  for a incident photon with central momentum  $\vec{k}$ . According to Eqs. (27, 26) and Eq. (19), it is clear that for fixed value of  $\vec{k}$ ,  $R_I(\vec{k})$  is a Lorentz function of the photon-atom detuning  $\Delta$ , and takes the maximum value under the condition  $\Delta = 2\xi(\cos k_x + \cos k_y) + \text{Re}[\Sigma(\vec{k})]$ . In Fig. 2 we illustrate  $R_I(\vec{k})$  with different periods of atoms. Here we choose the incident momentum  $(k_x, k_y) = (\pi/4, \pi/2)$ . The Lorentz-shape of the  $R_I(\vec{k})$  is clearly shown.

The single-peak behavior of  $R_I(\vec{k})$  can be explained by the following simple picture. Owing to the periodical structure of our system in the  $y$ -direction, in the single-photon scattering process the incident state  $|\vec{k}\rangle$  of the photon is coupled to the atomic spin-wave state

$$|S_{k_y}\rangle \equiv \sum_j e^{ik_y dj} |\tilde{e}_j\rangle. \quad (28)$$

As a result of this coupling, the effective energy of state  $|S_{k_y}\rangle$  is shifted from the bare value  $\omega_a$  to  $\omega_a + \text{Re}[\Sigma(\vec{k})]$ . Thus,  $R_I(\vec{k})$  takes the maximum value when the energy of the incident photon is resonance with the shifted energy  $\omega_a + \text{Re}[\Sigma(\vec{k})]$  of the spin-wave state, or the condition  $\epsilon_{\vec{k}} = \omega_a + \text{Re}[\Sigma(\vec{k})]$  is satisfied. It is pointed out that, the energy shift  $\text{Re}[\Sigma(\vec{k})]$  is related to the density  $d^{-1}$  of the atoms, and can be considered as the collective Lamb shift.

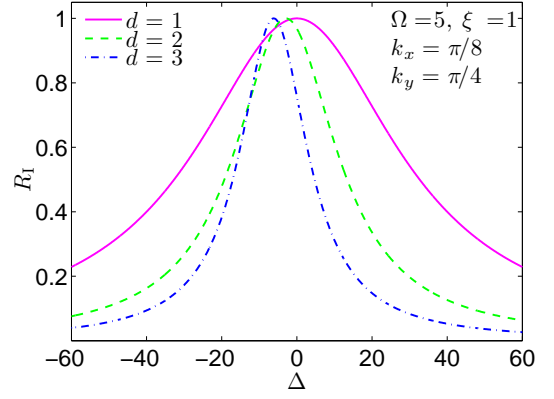


Figure 2: (Color online) The single-photon scattering probability  $R_I(\vec{k})$  as a function of photon-atom detuning  $\Delta$  with  $d = 1$  (purple solid line), 2 (green dash line) and 3 (blue dash-dotted line). Here we choose  $\vec{k} = (k_x, k_y) = (\pi/8, \pi/4)$  and  $\Omega = 5$ ,  $\xi = 1$ . The scattering probability  $R_I(\vec{k})$  has a Lorentz-shape profiles with Lamb shifts  $\text{Re}[\Sigma(\vec{k})]$  which are related to  $d$ .

The cooperative effect of the nuclear ensemble is reflected by the atomic-density-dependence of this collective Lamb shift and the width  $\text{Im}[\Sigma(\vec{k})]$  of the peak of  $R_I(\vec{k})$ .

In Fig. 3 (a, b) we plot the scattering probability  $R_I(\vec{k})$  as a function of  $\cos k_x$  for fixed values of  $\Delta$  and  $k_y$ . It is shown that, for some certain momentum  $k_x$ , we have  $R_I(\vec{k}) = 0$ . Namely, in these cases the photon is not scattered by the atoms. This phenomenon can be explained as follows. Due to the periodical structure of our system, for the incident photon with momentum  $\vec{k}$ , only the photon states  $|\vec{k}_l\rangle \equiv |k_{lx}, p_l(k_y)\rangle$ , which satisfies  $\epsilon_{\vec{k}_l} = \epsilon_{\vec{k}}$ , are involved in the scattering process. Eq. (13) shows that, the coupling between the atoms and the photon in the state  $|\vec{k}_l\rangle$  with a special  $l$  contributes the self-energy  $\Sigma_l(\vec{k})$ . With straightforward calculation, we can re-write  $\Sigma_l(\vec{k})$  defined in Eq. (13) as

$$\begin{aligned} \Sigma_l(\vec{k}) &= \frac{|\Omega|^2}{2\pi d} \int_{a_l}^{b_l} dx \frac{\rho(x)}{\epsilon_{\vec{k}} - x + i0^+} \\ &= -i \frac{|\Omega|^2}{2d} \rho(\epsilon_{\vec{k}}) + \frac{|\Omega|^2}{2\pi d} \text{P} \int_{a_l}^{b_l} dx \frac{\rho(x)}{\epsilon_{\vec{k}} - x}, \end{aligned} \quad (29)$$

where P means the principle integral,  $a_l = \epsilon_{(0, p_l(k_y))}$  and  $b_l = \epsilon_{(\pi, p_l(k_y))}$  are the lower and upper bounds of the energy  $\epsilon_{\vec{k}_l}$  with respect to the state  $|\vec{k}_l\rangle$ , respectively. In Eq. (29) we also have  $\rho(x) = 2dp(x)/dx$  with  $p(x) = \arccos[x - \omega_c + 2\xi \cos[p_l(k_y)]]$ . Usually, the self-energy is convergent for a system with bounded energy spectrum. However, in the present problem, when the energy of the incident photon is in the bound of the energies of the states  $|\vec{k}_l\rangle$  (i.e., the condition  $\epsilon_{\vec{k}} = a_l$  or  $\epsilon_{\vec{k}} = b_l$  is satisfied), the self-energy  $\Sigma_l(\vec{k})$  diverges according to Eq. (29). Furthermore, Eqs. (19, 26) and Eq. (27) yield

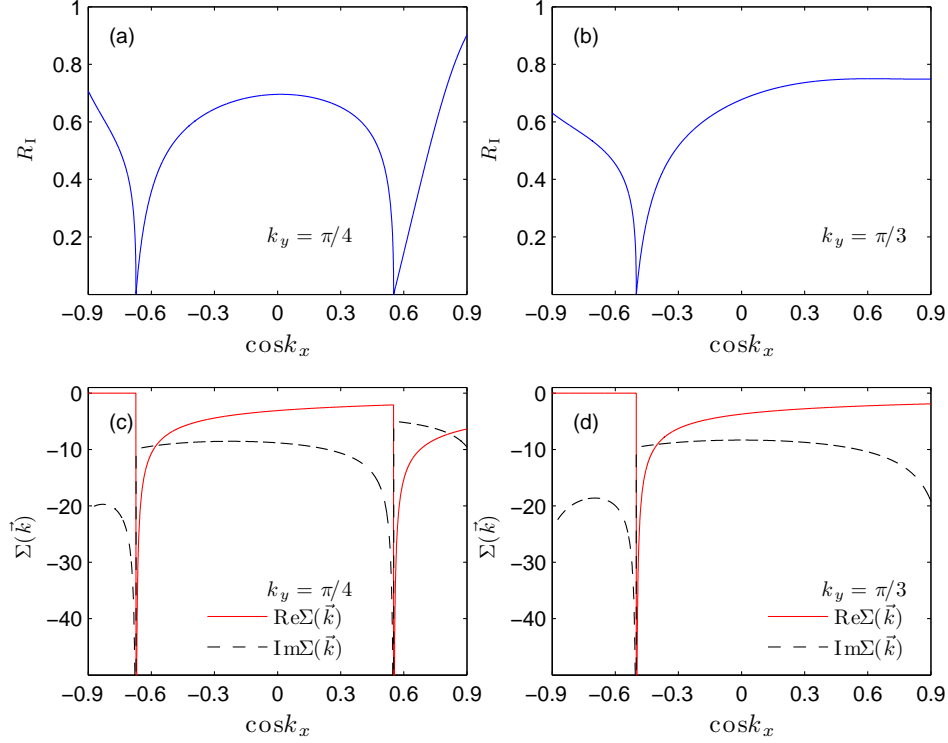


Figure 3: (Color online) (a) and (b): The single-photon scattering probability  $R_I(\vec{k})$  as a function of  $\cos k_x$  with  $k_y = \pi/4$  (a) and  $\pi/3$  (b). (c) and (d): the real part (red solid line) and imaginary part (black dashed line) of self-energy  $\Sigma(\vec{k})$  as functions of  $k_x$  with  $k_y = \pi/4$  (c) and  $\pi/3$  (d). Here, we also choose  $\Omega = 5$ ,  $\xi = 1$ ,  $\Delta = 0$  and  $d = 3$ .

$R_I(\vec{k}) = 0$  when the self-energy diverges.

The above observation is also verified by Eq. (16) which shows that  $\Sigma_l(\vec{k}) = \infty$  when  $|A_l| = 1$ . In addition, we illustrate the self-energy  $\Sigma(\vec{k})$  as a function of  $k_x$  in Fig. 3 (c, d). It is clearly shown that  $\Sigma(\vec{k}) = \infty$  when  $R_I(\vec{k}) = 0$ .

### III. SINGLE-PHOTON SCATTERING WITH TWO LAYERS OF ATOMS

In the above section we have studied the single-photon scattering on one layer of atoms in a 2D cavity array. We show that the single-photon scattering probability takes maximum value when the incident photon is resonant with the shifted atomic energy  $\omega_a + \text{Re}[\Sigma(\vec{k})]$ . In this section, we consider the single-photon scattering in the 2D cavity array with two layers of atoms located in the cavities at  $(x_1, dj)$  and  $(x_2, dj)$  with  $j = 0, \pm 1, \pm 2, \dots$  (Fig. 4). We will show that as a function of  $\Delta$ , the scattering probability has two peaks rather than a single one. The double-peak behavior results from the photon-induced effective coupling between atoms in different layers, and can be considered as an EIT-like phenomenon.

In the presence of two layers of atoms, the atom-photon

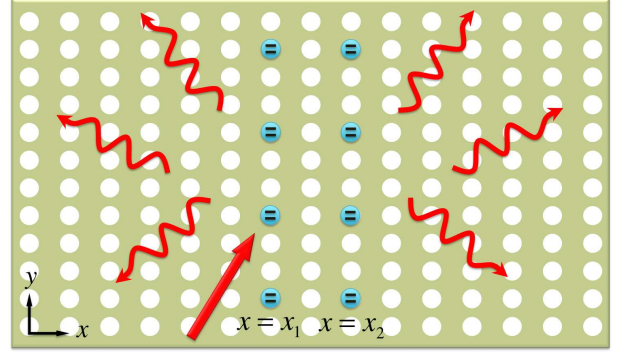


Figure 4: (Color online) The single-photon scattering by two layers of atoms in a 2D CCA. In this case the atoms are confined in two layers at  $x = x_1$  and  $x = x_2$ , with the same periodic  $d$ .

interaction reads

$$V = \sum_{s=1,2} \sum_{j=-\infty}^{\infty} (\Omega_s a_{(x_s, dj)} |e\rangle_j^{(s)} \langle g| + h.c.), \quad (30)$$

with  $|g(e)\rangle_j^{(s)}$  the ground (excited) state of the  $j$ -th atom in the  $s$ -th layer. The scattering state  $|\Psi^{(+)}\rangle$  can be

written as

$$|\Psi^{(+)}\rangle = |\phi\rangle|\tilde{g}\rangle + \sum_{s=1,2} \sum_{j=-\infty}^{\infty} \beta_j^{(s)} |\text{vac}\rangle |\tilde{e}_j^{(s)}\rangle, \quad (31)$$

with  $|\tilde{g}\rangle \equiv \prod_j |g\rangle_j^{(1)} |g\rangle_j^{(2)}$  the collective ground state of all atoms, and  $|\tilde{e}_j^{(s)}\rangle$  ( $s = 1, 2$ ) defined as  $|\tilde{e}_j^{(s)}\rangle \equiv |e\rangle_j^{(s)} \prod_{l \neq j} |g\rangle_l^{(s)} \prod_j |g\rangle_j^{(3-s)}$  denotes the state in which only the  $j$ -th atom in the  $s$ -th layer is excited. Moreover, the translation invariance along the  $y$ -axis leads to the result  $\beta_j^{(s)} = \beta^{(s)} \exp[i(k_x x_s + k_y d j)]$ . Substituting this result into the Lippmann-Schwinger equation, we find that the  $j$ -independent coefficients  $\beta^{(1,2)}$  have similar expressions with the parameter  $\beta$  in Eq. (11), and can be written as

$$\begin{pmatrix} \beta^{(1)} \\ \beta^{(2)} \end{pmatrix} = \frac{1}{2\pi [\omega_{\vec{k}} - \omega_a - \Sigma(\vec{k})]} \begin{pmatrix} \Omega_1 \\ \Omega_2 \end{pmatrix} \quad (32)$$

Here the self-energy  $\Sigma(\vec{k})$  is now a  $2 \times 2$  matrix

$$\Sigma(\vec{k}) = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}, \quad (33)$$

with elements  $\Sigma_{ij}$  ( $i, j = 1, 2$ ) given by

$$\Sigma_{ij} = \frac{\Omega_i \Omega_j^*}{2\pi d} \sum_{l=0}^{d-1} \int_{-\pi}^{\pi} dq_x \frac{\exp[-i(k_x - q_x)(x_i - x_j)]}{\epsilon_{\vec{k}} - \epsilon_{[q_x, p_l(k_y)]} + i0^+}. \quad (34)$$

It is clear that Eq. (32) can be solved straightforwardly and we have

$$\beta^{(s)} = \frac{\Omega_s}{2\pi(\Sigma_+ - \Sigma_-)} \left( \frac{\Sigma_+ - J_s}{\Delta_{\vec{k}} - \Sigma_+} - \frac{\Sigma_- - J_s}{\Delta_{\vec{k}} - \Sigma_-} \right), \quad (35)$$

for  $s = 1, 2$ . Here  $\Delta_{\vec{k}} = \omega_{\vec{k}} - \omega_a$  and  $J_s$  is defined as

$$J_s = \Sigma_{(3-s), (3-s)} - \frac{\Omega_{(3-s)}}{\Omega_s} \Sigma_{s, (3-s)}. \quad (36)$$

In Eq. (35),  $\Sigma_{\pm}(\vec{k})$  is the eigenvalue of matrix  $\Sigma(\vec{k})$  and takes the form

$$\Sigma_{\pm} = \frac{1}{2} \left[ \Sigma_{11} + \Sigma_{22} \pm \sqrt{(\Sigma_{11} - \Sigma_{22})^2 + 4\Sigma_{12}\Sigma_{21}} \right]. \quad (37)$$

With these results, we can derive the expressions of the scattering state  $|\Psi^{(+)}\rangle$ , and the on-shell element  $t(\vec{k}' \leftarrow \vec{k})$  of the  $T$ -matrix is

$$t(\vec{k}' \leftarrow \vec{k}) = u_{\text{II}}(\vec{k}) \sum_l \delta[k'_y - p_l(k_y)]. \quad (38)$$

Now the coefficient function  $u_{\text{II}}(\vec{k})$  is given by

$$u_{\text{II}}(\vec{k}) = \frac{1}{d} \sum_{s=1,2} e^{-i(k'_x - k_x)x_s} \Omega_s^* \beta^{(s)}. \quad (39)$$

Similar as the above section, for an incident photon with central momentum  $\vec{k}$ , the scattering probability  $R_{\text{II}}(\vec{k})$  can be expressed in terms of the  $T$ -matrix element, and the straightforward calculation gives

$$R_{\text{II}}(\vec{k}) = 2 \sum_{l=1}^d \frac{|u_{\text{II}}(\vec{k})|^2}{4\xi^2 |\sin k_x \sin k_{lx}|}, \quad (40)$$

with  $k_{lx}$  determined by the equation  $\epsilon_{\vec{k}} = \epsilon_{[k_{lx}, p_l(k_y)]}$ .

Now we investigate the behavior of the scattering probability  $R_{\text{II}}(\vec{k})$  with respect to the bare detuning  $\Delta$ . With Eq. (39) and Eq. (35), we find that  $R_{\text{II}}(\vec{k})$  takes local maximum values when the condition  $\text{Re}[\Delta_{\vec{k}} - \Sigma_{\pm}(\vec{k})] = 0$  is satisfied. Namely, unlike the single-peak behavior shown in Eq. (11) and Fig. (2) for the single-layer case, in our current system  $R_{\text{II}}(\vec{k})$  has two peaks around the positions

$$\Delta_{\pm} \equiv \text{Re}[\Sigma_{\pm}] + 2\xi(\cos k_x + \cos k_y). \quad (41)$$

Furthermore, in the two-layer case, the cooperative effect of the nuclear ensemble is reflected by the atomic density  $d^{-1}$  dependence of the Lamb shifts  $\text{Re}[\Sigma_{\pm}(\vec{k})]$ , the peak widths  $\text{Im}[\Sigma_{\pm}(\vec{k})]$  and the distance  $\text{Re}[\Sigma_+(\vec{k}) - \Sigma_-(\vec{k})]$  between the two peaks.

This observation is verified by our exact numerical calculation for Eq. (40). In Fig. (5), we illustrate  $R_{\text{II}}(\vec{k})$  for the one- and two-layer cases. The double-peak behavior of  $R_{\text{II}}(\vec{k})$  at  $\Delta = \Delta_{\pm}$  for the two-layer case is clearly shown in Fig. (5a-5c). It is pointed out that, such a behavior essentially has the same physical mechanism with that of the atomic susceptibility in an EIT system. In typical EIT system, two internal states of the  $\Lambda$ -type atom are dressed with the control laser beam, and form two non-degenerate dressed states. Then the atomic susceptibility takes local maximum value when the incident photon is resonance with one of these two states. In our problem, the incident state  $|\vec{k}\rangle$  of the photon is coupled to two quasi spin-wave states

$$|S_{k_y}^{(1,2)}\rangle \equiv \sum_j e^{ik_y d j} |\tilde{e}_j^{(1,2)}\rangle \quad (42)$$

with respect to the excitations of the atoms in the 1st and 2nd layers, respectively. These two states have the same bare energy  $\omega_a$ . Nevertheless, due to the atom-photon coupling,  $|S_{k_y}^{(1)}\rangle$  and  $|S_{k_y}^{(2)}\rangle$  are effectively coupled with each other via the non-diagonal elements  $\Sigma_{12}$  and  $\Sigma_{21}$  of the self-energy matrix  $\Sigma(\vec{k})$ , and form two dressed states [Fig. (6)]. These two dressed states have different effective energies  $\omega_a + \Sigma_{\pm}$ . The single-photon scattering probability  $R_{\text{II}}(\vec{k})$  takes local maximum values when the incident photon is resonant with one of these two dressed states. Therefore, the  $R_{\text{II}}(\vec{k})$  presents an EIT-like behavior which is illustrated by its two peaks at  $\Delta = \Delta_{\pm}$ .

In the end, we emphasize that the above analysis is a qualitative one which provides a physical picture for the

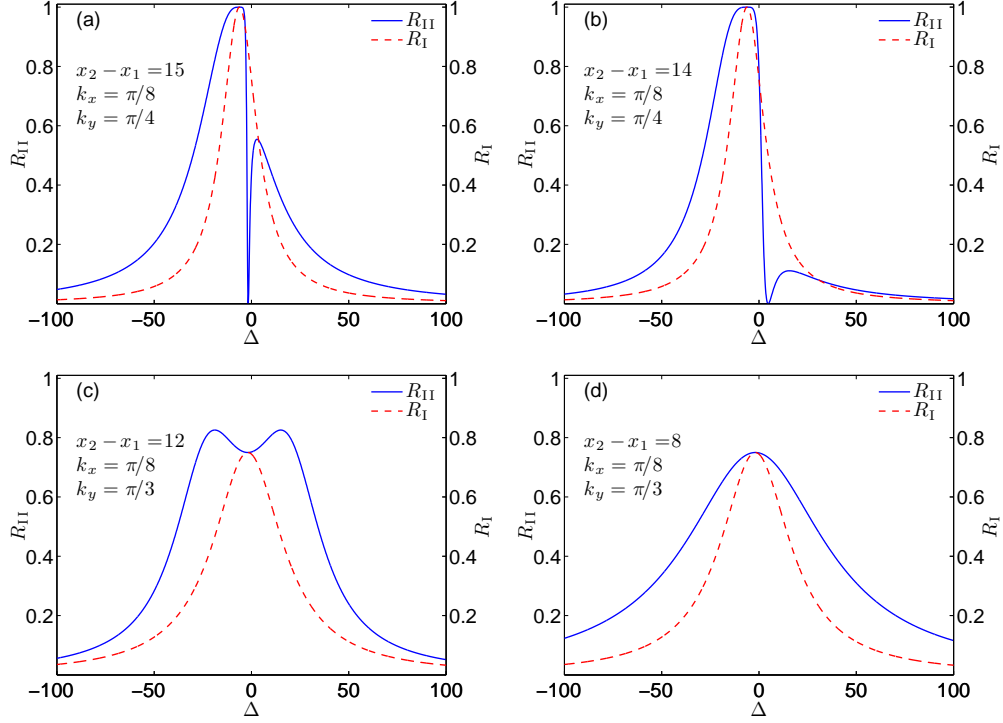


Figure 5: (Color online) The single-photon scattering probability  $R_{II}(\vec{k})$  for the two-layer case (solid blue line). The peak behavior of  $R_{II}(\vec{k})$  at  $\Delta = \Delta_{\pm}$  is illustrated for  $x_2 - x_1 = 15$ ,  $(k_x, k_y) = (\pi/8, \pi/4)$  (a),  $x_2 - x_1 = 14$ ,  $(k_x, k_y) = (\pi/8, \pi/4)$  (b),  $x_2 - x_1 = 12$ ,  $(k_x, k_y) = (\pi/8, \pi/3)$  (c) and  $x_2 - x_1 = 8$ ,  $(k_x, k_y) = (\pi/8, \pi/3)$  (c). Here we choose  $\Omega_1 = \Omega_2 = 5$ ,  $\xi = 1$  and  $d = 3$ . As a comparison, we also plot the scattering probability  $R_I(k)$  for the single-layer cases with the same parameters (dashed red line).

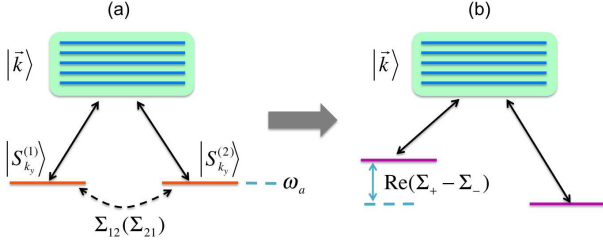


Figure 6: (a): In the two-layer case, the two quasi spin-wave states  $|S_{k_y}^{(1)}\rangle$  and  $|S_{k_y}^{(2)}\rangle$ , which have the same bare energy  $\omega_a$ , are coupled to the continuous spectrum of the photonic states. Due to this atom-photon coupling,  $|S_{k_y}^{(1)}\rangle$  and  $|S_{k_y}^{(2)}\rangle$  are effectively coupled with each other by the non-diagonal terms  $\Sigma_{12}$  and  $\Sigma_{21}$  of the self-energy matrix  $\Sigma(\vec{k})$ . (b): As a result of this effective coupling,  $|S_{k_y}^{(1)}\rangle$  and  $|S_{k_y}^{(2)}\rangle$  form two dressed states with different effective energies  $\omega_a + \Sigma_{\pm}$ .

EIT-like behavior of the single-photon scattering probability  $R_{II}(\vec{k})$ . On the other hand, due to the complicated expressions of the *complex* self-energy matrix  $\Sigma(\vec{k})$ , in different parameter regions, the widths and heights of the two peaks of  $R_{II}(\vec{k})$  can be quite different, as shown in Fig. (5). Moreover, in some special cases the peak-widths could be larger than the inter-peak distance, and

thus the two peaks emerge into a single one Fig. (5d).

#### IV. CONCLUSION

In this paper, we study the single-photon scattering by one or two layers of identical atoms in a 2D CCA. We obtain the exact analytical expression for the single-photon scattering probability, and explore the relevant cooperative effects. In the case of one atomic layer, a Lorentz-type peak of the scattering probability implies the collective Lamb shift, while in the two-layer case an EIT-like double-peak behavior appears, and the cooperative effect of the ensemble is reflected by the atomic-density-dependence of the position, width and distance of the two peaks. Our approach fully characterizes the quantum nature of the photon and the internal structure of the nucleus, and will be helpful for the study of the 2D photonic quantum devices and X-ray quantum optics from the perspective of quantum scattering.

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### Appendix: single-photon outgoing wave packets and probability $P_l$

In this appendix derive the outgoing wave function of a single-photon scattered by the atoms, and prove Eq. (26) for the probability  $P_l$ . Substituting Eqs. (18, 24) into Eqs. (22, 23), it is easy to find that after the scattering process, the incident wave packet splits into  $(2d - 1)$  different ones. Namely, the out-going wave function in Eq. (23) is given by

$$\phi^{(out)}(\vec{k}) = \sum_{l=-d}^d \phi_l^{(out)}(\vec{k}). \quad (43)$$

Here the function  $\phi_l^{(out)}(\vec{k})$  is defined as

$$\phi_0^{(out)}(\vec{k}) = \frac{u(\vec{k})\phi^{(in)}(\vec{k})}{2\xi |\sin k_x|}, \quad (44)$$

for  $l = 0$  and

$$\phi_l^{(out)}(\vec{k}) = \frac{u[\vec{f}_l(\vec{k})]\phi^{(in)}[\vec{f}_l(\vec{k})]}{2\xi |\sin f_{lx}(\vec{k})|} \delta_{\text{sign}(k_x), \text{sign}(l)} \quad (45)$$

with the function  $\vec{f}_l(\vec{k}) = [f_{lx}(\vec{k}), f_{ly}(\vec{k})]$  defined as

$$f_{lx}(\vec{k}) = \arccos \left\{ \cos k_x + \cos k_y - \cos [f_{ly}(\vec{k})] \right\}, \quad (46)$$

$$f_{ly}(\vec{k}) = (k_y + \pi + \frac{2\pi|l|}{d}) \bmod [2\pi] - \pi, \quad (47)$$

and the Kronecker symbol  $\delta_{i,j}$  satisfies  $\delta_{i,j} = 1$  for  $i = j$  and  $\delta_{i,j} = 0$  for  $i \neq j$ . In Eqs. (44) and (45), we

have  $u(\vec{k}) = u_I(\vec{k})$  for the case with one layer of atoms and  $u(\vec{k}) = u_{II}(\vec{k})$  for the two-layer case. Then it is easy to see that, when the incident wave packet  $\phi^{(in)}(\vec{k})$  sharply peaks at a specific momentum  $\vec{k}_0 = (k_{0x}, k_{0y})$ , the out-put wave packet  $\phi_l^{(out)}(\vec{k})$  sharply peaks at the momentum  $\vec{k}_l = [k_{lx}, p_l(k_{0y})]$  which satisfies  $\epsilon_{\vec{k}_l} = \epsilon_{\vec{k}_0}$  and  $\text{sign}(k_{lx}) = \text{sign}(l)$ .

Now we calculate the probability  $P_l = \int |\phi_l^{(out)}(\vec{k})|^2 d\vec{k}$  for the cases with  $l \neq 0$ . Apparently, we have

$$P_l = \int \left| \frac{u[\vec{f}_l(\vec{k})]\phi^{(in)}[\vec{f}_l(\vec{k})]}{2\xi |\sin f_{lx}(\vec{k})|} \delta_{\text{sign}(k_x), \text{sign}(l)} \right|^2 d\vec{k} \quad (48)$$

We define  $g_{lx} = f_{lx}(\vec{k})$  and  $g_{ly} = f_{ly}(\vec{k})$ . Then using the fact that  $\phi^{(in)}(\vec{k})$  sharply peaks at a specific momentum  $\vec{k}_0$  and the relations

$$dk_x = \left| \frac{\sin g_{lx}}{\sin k_x} \right| dg_{lx}, dk_y = dk_{ly}, \quad (49)$$

we have

$$\begin{aligned} P_l &\approx \int \left| \frac{u[\vec{f}_l(\vec{k})]\phi^{(in)}[\vec{f}_l(\vec{k})]}{2\xi |\sin f_{lx}(\vec{k})|} \right|^2 d\vec{k} \\ &\approx \frac{|u(\vec{k}_0)|^2}{4\xi^2 |\sin k_{0x} \sin k_{lx}|} \int |\phi^{(in)}(g_{lx}, g_{ly})|^2 dg_{lx} dg_{ly} \\ &= \frac{|u(\vec{k}_0)|^2}{4\xi^2 |\sin k_{0x} \sin k_{lx}|}. \end{aligned} \quad (50)$$

Then we have proved Eq. (26).

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